

## THE LMA MSW SOLUTION OF THE SOLAR NEUTRINO PROBLEM, INVERTED NEUTRINO MASS HIERARCHY AND REACTOR NEUTRINO EXPERIMENTS

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**Abstract.** In the context of three-neutrino oscillations, we study the possibility of using antineutrinos from nuclear reactors to explore the  $10^{-4} \text{ eV}^2 < \Delta m_{\odot}^2 \lesssim 8 \times 10^{-4} \text{ eV}^2$  region of the LMA MSW solution of the solar neutrino problem. The KamLAND experiment is not expected to determine  $\Delta m_{\odot}^2$  if the latter happens to lie in the indicated region. By analysing both the total event rate suppression and the energy spectrum distortion caused by  $\bar{\nu}_e$  oscillations in vacuum, we show that the optimal baseline of such an experiment is  $L \sim (20 - 10) \text{ km}$ . Furthermore, the same experiment might be used to try to distinguish between the two possible types of neutrino mass spectrum - with normal or with inverted hierarchy, by exploring the effect of interference between the atmospheric- and solar-  $\Delta m^2$  driven oscillations. The latter modifies in a characteristic way the energy spectrum of detected events. Distinguishing between the two types of neutrino mass spectrum requires, however, a high precision determination of the atmospheric  $\Delta m^2$ , a sufficiently large  $\sin^2 \theta$  and a non-maximal  $\sin^2 2\theta_{\odot}$ , where  $\theta$  and  $\theta_{\odot}$  are the mixing angles respectively limited by the CHOOZ and Palo Verde data and characterizing the solar neutrino oscillations.

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# 1 Introduction

In recent years the experiments with solar and atmospheric neutrinos collected strong evidences in favor of the existence of oscillations between the flavour neutrinos,  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$ . Further progress in our understanding of the neutrino mixing and oscillations requires, in particular, precise measurements of the parameters entering into the oscillation probabilities - the neutrino mass-squared differences and mixing angles, and the reconstruction of the neutrino mass spectrum.

The atmospheric neutrino data can be explained by dominant  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  oscillations, characterized by large, possibly maximal, mixing, and a mass squared difference,  $\Delta m_{atm}^2$ , having a value in the range [1] (99% C.L.):

$$1.3 \times 10^{-3} \text{eV}^2 \lesssim |\Delta m_{atm}^2| \lesssim 5 \times 10^{-3} \text{eV}^2. \quad (1)$$

The first results from the Sudbury Neutrino Observatory (SNO) [2], combined with the mean event rate data from the Super-Kamiokande (SK) experiment [3], provide a very strong evidence for oscillations of the solar neutrinos [4, 5, 6, 7, 8, 9, 10]. Global analyses of the solar neutrino data, including the SNO results and the SK data on the  $e^-$ -spectrum and day-night asymmetry, show that the data favor the large mixing angle (LMA) MSW solution of the solar neutrino problem, with the corresponding neutrino mixing parameter  $\sin^2 2\theta_\odot$  and mass-squared difference  $\Delta m_\odot^2$  lying in the regions (99.73% C.L.):

$$2 \times 10^{-5} \text{eV}^2 \lesssim \Delta m_\odot^2 \lesssim 8 \times 10^{-4} \text{eV}^2 \quad (2)$$

$$0.6 \leq \sin^2 2\theta_\odot \leq 1. \quad (3)$$

The best fit value of  $\Delta m_\odot^2$  found in the independent analyses [5, 6, 7, 9] is spread in the interval  $(4.3 - 6.3) \times 10^{-5} \text{eV}^2$ . The results obtained in [5, 6, 7, 9] show that values of  $\Delta m_\odot^2 > 10^{-4} \text{eV}^2$  are allowed already at 90% C.L.

Important constraints on the oscillations of electron (anti-)neutrinos, which play a significant role in our current understanding of the possible patters of oscillations of the three flavour neutrinos and anti-neutrinos, were obtained in the CHOOZ and Palo Verde disappearance experiments with reactor  $\bar{\nu}_e$  [11, 12]. The CHOOZ and Palo Verde experiments were sensitive to values of neutrino mass squared difference  $\Delta m^2 \gtrsim 10^{-3} \text{eV}^2$ , which includes the corresponding atmospheric neutrino region, eq. (1). No disappearance of the reactor  $\bar{\nu}_e$  was observed. Performing a two-neutrino oscillation analysis, the following rather stringent upper bound on the value of the corresponding mixing angle,  $\theta$ , was obtained by the CHOOZ collaboration <sup>1</sup> [11] at 95% C.L. for  $\Delta m^2 \geq 1.5 \times 10^{-3} \text{eV}^2$ :

$$\sin^2 \theta < 0.09. \quad (4)$$

The precise upper limit in eq. (4) is  $\Delta m^2$ -dependent: it is a decreasing function of  $\Delta m^2$  as  $\Delta m^2$  increases up to  $\Delta m^2 \simeq 6 \cdot 10^{-3} \text{eV}^2$  with a minimum value  $\sin^2 \theta \simeq 10^{-2}$ . The upper limit becomes an increasing function of  $\Delta m^2$  when the latter increases further up to  $\Delta m^2 \simeq 8 \cdot 10^{-3} \text{eV}^2$ , where  $\sin^2 \theta < 2 \cdot 10^{-2}$ . Somewhat weaker constraints on  $\sin^2 \theta$  have been obtained by the Palo Verde collaboration [12]. In the future,  $\sin^2 \theta$  might be further constrained or determined, e.g., in long baseline neutrino oscillation experiments [13].

The long baseline experiment with reactor  $\bar{\nu}_e$  KamLAND [14] has been designed to test the LMA MSW solution of the solar neutrino problem. This experiment is planned to provide a rather precise measurement of  $\Delta m_\odot^2$  and  $\sin^2 2\theta_\odot$ . Due to the long baseline of the experiment,  $L \sim 180 \text{ km}$ , however,  $\Delta m_\odot^2$  can be determined with a relatively good precision only if  $\Delta m_\odot^2 \lesssim 10^{-4} \text{eV}^2$ .

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<sup>1</sup>The possibility of large  $\sin^2 \theta > 0.9$  which is admitted by the CHOOZ data alone is incompatible with the neutrino oscillation interpretation of the solar neutrino deficit (see, e.g., [15, 16])

The explanation of both the atmospheric and solar neutrino data in terms of neutrino oscillations requires, as is well-known, the existence of 3-neutrino mixing in the weak charged lepton current:

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL}, \quad (5)$$

where  $\nu_{lL}$ ,  $l = e, \mu, \tau$ , are the three left-handed flavour neutrino fields,  $\nu_{jL}$  is the left-handed field of the neutrino  $\nu_j$  having a mass  $m_j > 0$  and  $U$  is a  $3 \times 3$  unitary mixing matrix - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [17, 18]. The three neutrino masses  $m_{1,2,3}$  can obey the so-called normal hierarchy (NH) relation  $m_1 < m_2 < m_3$ , or that of the inverted hierarchy (IH) type,  $m_3 < m_1 < m_2$ . Thus, in order to reconstruct the neutrino mass spectrum in the case of 3-neutrino mixing, it is necessary to establish, in particular, which of the two possible types of neutrino mass spectrum is actually realized. This information is particularly important for the studies of a number of fundamental issues related to lepton mixing, as like the possible Majorana nature of massive neutrinos, which can manifest itself in the existence of neutrino-less double  $\beta$ -decay (see, e.g., [19, 20]). It would also constitute a critical test for theoretical models of fermionic mass matrices and flavor physics in general.

It would be possible to determine whether the neutrino mass spectrum is with normal or inverted hierarchy in terrestrial neutrino oscillation experiments with a sufficiently long baseline, so that the neutrino oscillations take place in the Earth and the Earth matter effects in the oscillations are non-negligible [21, 22, 23]. The ambiguity regarding the type of the neutrino mass spectrum might be resolved by the MINOS experiment [13], although on the baseline of this experiment the matter effects are relatively small [21]. This might be done in an experiment with atmospheric neutrinos, utilizing a detector with a sufficiently good muon charge discrimination [24]. The experiments at neutrino factories would be particularly suitable for the indicated purpose [22, 23].

In this paper, in the context of three-neutrino oscillations, we study the possibility of using anti-neutrinos from nuclear reactors to explore the  $\Delta m_{\odot}^2 > 10^{-4} \text{ eV}^2$  region of the LMA MSW solution. Such an experiment might be of considerable interest if, in particular, the results of the KamLAND experiment will confirm the validity of the LMA-MSW solution of the solar neutrino problem, but will allow to obtain only a lower bound on  $\Delta m_{\odot}^2$  due to the fact that  $\Delta m_{\odot}^2 > 10^{-4} \text{ eV}^2$ . We determine the optimal baseline of the possible experiment with reactor  $\bar{\nu}_e$ , which would provide a precise measurement of  $\Delta m_{\odot}^2$  in the region  $10^{-4} \text{ eV}^2 < \Delta m_{\odot}^2 \lesssim 8 \times 10^{-4} \text{ eV}^2$ . Furthermore, the same experiment might be used to try to distinguish between the two types of neutrino mass spectrum - with normal or with inverted hierarchy. This might be done by exploring the effect of interference between the amplitudes of neutrino oscillations, driven by the solar and atmospheric  $\Delta m^2$ , i.e., by  $\Delta m_{\odot}^2$  and  $\Delta m_{atm}^2$ . The indicated effect could be relevant for  $\Delta m_{\odot}^2 > 10^{-4} \text{ eV}^2$ . Distinguishing between the two possible types of neutrino mass spectrum requires a measurement of  $\Delta m_{atm}^2$  with very high precision,  $\sin^2 2\theta_{\odot} \neq 1.0$ , e.g.,  $\sin^2 2\theta_{\odot} \lesssim 0.9$ , and a sufficiently large value of the angle  $\theta$ , which for  $\Delta m_{\odot}^2 \ll \Delta m_{atm}^2$  controls, e.g., the oscillations of the atmospheric  $\nu_e$  and  $\bar{\nu}_e$  and is constrained by the CHOOZ and Palo Verde data.

## 2 The $\bar{\nu}_e$ Survival Probability

We shall assume in what follows that the 3-neutrino mixing described by eq. (5) takes place. We shall number (without loss of generality) the neutrinos with definite mass in vacuum  $\nu_j$ ,  $j = 1, 2, 3$ , in such a way that their masses obey  $m_1 < m_2 < m_3$ . Then the cases of NH and IH neutrino mass spectrum differ, in particular, by the relation between the mixing matrix elements  $|U_{ej}|$ ,  $j = 1, 2, 3$ , and the mixing angles  $\theta_{\odot}$  and  $\theta$  (see further). With the indicated choice one has  $\Delta m_{jk}^2 > 0$  for  $j > k$ . Let us emphasize that we do not assume any of the relations  $m_1 \ll m_2 \ll m_3$ , or  $m_1 \lesssim m_2 \ll m_3$ , or  $m_1 \ll m_2 \cong m_3$ , to be valid in what follows.

Under the conditions of the experiment we are going to discuss, which must have a baseline  $L$  considerably shorter than the baseline  $\sim 180$  km of the KamLAND experiment, the reactor  $\bar{\nu}_e$  oscillations will not be affected by Earth matter effects when the  $\bar{\nu}_e$  travel between the source (reactor) and the detector. If 3-neutrino mixing takes place, eq. (5), the  $\bar{\nu}_e$  would take part in 3-neutrino oscillations in vacuum on the way to the detector.

We shall obtain next the expressions for the reactor  $\bar{\nu}_e$  survival probability of interest in terms of measurable quantities for the two types of neutrino mass spectrum. In the case of normal hierarchy between the neutrino masses we have:

$$\Delta m_{\odot}^2 = \Delta m_{21}^2, \quad (6)$$

and

$$|U_{e1}| = \cos \theta_{\odot} \sqrt{1 - |U_{e3}|^2}, \quad |U_{e2}| = \sin \theta_{\odot} \sqrt{1 - |U_{e3}|^2}, \quad (7)$$

where

$$\theta_{\odot} = \theta_{12}, \quad |U_{e3}|^2 = \sin^2 \theta \equiv \sin^2 \theta_{13}, \quad (8)$$

$\theta_{12}$  and  $\theta_{13}$  being two of the three mixing angles in the standard parameterization of the PMNS matrix (see, e.g., [16]). Note that  $|U_{e3}|^2$  is constrained by the CHOOZ and Palo Verde results. It is not difficult to derive the expression for the  $\bar{\nu}_e$  survival probability in the case under discussion:

$$\begin{aligned} P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - 2 \sin^2 \theta \cos^2 \theta \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E_{\nu}} \right) \\ &- \frac{1}{2} \cos^4 \theta \sin^2 2\theta_{\odot} \left( 1 - \cos \frac{\Delta m_{\odot}^2 L}{2 E_{\nu}} \right) \\ &+ 2 \sin^2 \theta \cos^2 \theta \sin^2 \theta_{\odot} \left( \cos \left( \frac{\Delta m_{31}^2 L}{2 E_{\nu}} - \frac{\Delta m_{\odot}^2 L}{2 E_{\nu}} \right) - \cos \frac{\Delta m_{31}^2 L}{2 E_{\nu}} \right), \end{aligned} \quad (9)$$

where  $E_{\nu}$  is the neutrino energy and we have made use of eqs. (6), (7) and (8).

If the neutrino mass spectrum is with inverted hierarchy one has (see, e.g., [25, 20, 16]):

$$\Delta m_{\odot}^2 = \Delta m_{32}^2, \quad (10)$$

and

$$|U_{e2}| = \cos \theta_{\odot} \sqrt{1 - |U_{e1}|^2}, \quad |U_{e3}| = \sin \theta_{\odot} \sqrt{1 - |U_{e1}|^2}. \quad (11)$$

The mixing matrix element constrained by the CHOOZ and Palo Verde data is now  $|U_{e1}|^2$ :

$$|U_{e1}|^2 = \sin^2 \theta. \quad (12)$$

The expression for the  $\bar{\nu}_e$  survival probability can be written in the form [26]:

$$\begin{aligned} P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - 2 \sin^2 \theta \cos^2 \theta \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E_{\nu}} \right) \\ &- 12 \cos^4 \theta \sin^2 2\theta_{\odot} \left( 1 - \cos \frac{\Delta m_{\odot}^2 L}{2 E_{\nu}} \right) \\ &+ 2 \sin^2 \theta \cos^2 \theta \cos^2 \theta_{\odot} \left( \cos \left( \frac{\Delta m_{31}^2 L}{2 E_{\nu}} - \frac{\Delta m_{\odot}^2 L}{2 E_{\nu}} \right) - \cos \frac{\Delta m_{31}^2 L}{2 E_{\nu}} \right). \end{aligned} \quad (13)$$

Several comments concerning the expressions for the  $\bar{\nu}_e$  survival probability, eqs. (9) and (13), follow. In the first lines in the right-hand side of eqs. (9) and (13), the oscillations of the electron (anti-)neutrino driven by the “atmospheric”  $\Delta m_{31}^2$  are accounted for. The CHOOZ and Palo Verde experiments are primarily sensitive to this term and their results limit  $\sin^2 \theta$ . The second lines in the expressions in eqs. (9) and (13) contain the solar neutrino oscillation parameters. This is the term KamLAND should be most sensitive to. For  $\Delta m_{\odot}^2 \ll \Delta m_{31}^2 = \Delta m_{atm}^2$ ,  $\Delta m_{\odot}^2 \lesssim 10^{-4} \text{ eV}^2$ , only one of the indicated two terms leads to an oscillatory dependence of the  $\bar{\nu}_e$  survival probability for the ranges of  $L/E_\nu$  characterizing the CHOOZ and Palo Verde, and the KamLAND experiments: on the source-detector distance  $L$  of the CHOOZ and Palo Verde experiments the oscillations due to  $\Delta m_{\odot}^2$  cannot develop, while on the distance(s) traveled by the  $\bar{\nu}_e$  in the KamLAND experiment  $\Delta m_{atm}^2$  causes fast oscillations which average out and are not predicted to lead, e.g., to specific spectrum distortions of the KamLAND event rate.

The terms in the third lines in eqs. (9) and (13) are not present in any two-neutrino oscillation analysis. They represent interference terms between the amplitudes of neutrino oscillations, driven by the solar and atmospheric neutrino mass squared differences. The term in eq. (9) is proportional to  $\sin^2 \theta_{\odot}$ , while the corresponding term in eq. (13) is proportional to  $\cos^2 \theta_{\odot}$  [26]. This is the only difference between  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and  $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ , that can be used to distinguish between the two cases of neutrino mass spectrum in an experiment with reactor  $\bar{\nu}_e$ . Obviously, if  $\cos 2\theta_{\odot} = 0$ , we have  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and the two types of spectrum would be indistinguishable in the experiments under discussion. For vanishing  $\sin^2 \theta$ , only the terms in the second line of eqs. (9) and (13) survive, and the two-neutrino mixing formula for solar neutrino oscillations in vacuum is exactly reproduced.

Let us discuss next the ranges of values the different oscillation parameters, which enter into the expressions for the probabilities of interest  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and  $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ , can take. The allowed region of values of  $\Delta m_{31}^2$ ,  $\Delta m_{\odot}^2$ ,  $\sin^2 \theta_{\odot}$  and  $\theta$  should be determined in a global 3-neutrino oscillation analysis of the solar, atmospheric and reactor neutrino oscillation data, in which, in particular,  $\Delta m_{\odot}^2$  should be allowed to take values in the LMA solution region, including the interval  $\Delta m_{\odot}^2 \sim (1.0 - 6.0) \times 10^{-4} \text{ eV}^2$ . Such an analysis is lacking in the literature. However, as was shown in [27], a global analysis of the indicated type would not change essentially the results for the LMA MSW solution we have quoted <sup>2</sup> in eqs. (2) and (3) as long as  $\Delta m_{31}^2 \gtrsim 1.5 \times 10^{-3} \text{ eV}^2$ . The reason is that for  $\Delta m_{31}^2 \gtrsim 1.5 \times 10^{-3} \text{ eV}^2$  and  $\Delta m_{\odot}^2 \lesssim 6.0 \times 10^{-4} \text{ eV}^2$ , the solar  $\nu_e$  survival probability, which determines the level of suppression of the solar neutrino flux and plays a major role in the analyses of the solar neutrino data, depends very weakly on (i.e., is practically independent of)  $\Delta m_{31}^2$ . Thus,  $\Delta m_{\odot}^2$  and  $\theta_{\odot}$  are uniquely determined by the solar neutrino and CHOOZ and Palo Verde data, independently of the atmospheric neutrino data and of the type of the neutrino mass spectrum. The CHOOZ and Palo Verde data lead to an upper limit on  $\Delta m_{\odot}^2$  in the LMA MSW solution region; for  $\Delta m_{\odot}^2 \lesssim 1.0 \times 10^{-4} \text{ eV}^2$ , the CHOOZ and solar neutrino data imply the upper limit on  $\sin^2 \theta$  given in eq. (4). For  $\Delta m_{\odot}^2 \sim (2.0 - 6.0) \times 10^{-4} \text{ eV}^2$  of interest, the upper limit on  $\sin^2 \theta$  as a function of  $\Delta m_{31}^2 \gtrsim 10^{-3} \text{ eV}^2$  for given  $\Delta m_{\odot}^2$  and  $\sin^2 2\theta_{\odot}$  is somewhat more stringent [26].

Would a global 3-neutrino oscillation analysis of the solar, atmospheric and reactor neutrino oscillation data lead to drastically different results for  $\Delta m_{31}^2$  in the two cases of normal and inverted neutrino mass hierarchy? Our preliminary analysis shows that given the existing atmospheric neutrino data from the Super-Kamiokande experiment, such an analysis i) would not be able to discriminate between the two cases of neutrino mass spectrum, and ii) would give essentially the same allowed region for  $\Delta m_{31}^2$  in the two cases of neutrino mass spectrum. We expect the regions of allowed values of the mixing angle  $\theta_{atm}$ , which controls the dominant atmospheric  $\nu_{\mu} \rightarrow \nu_{\tau}$  and  $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\tau}$  oscillations,

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<sup>2</sup>Let us note that the LMA MSW solution values of  $\Delta m_{\odot}^2$  and  $\theta_{\odot}$  we quote in eqs. (2) and (3) were obtained by taking into account the CHOOZ and Palo Verde limits as well.

to differ somewhat in the two cases. Note, however, that this mixing angle does not enter the expression for the  $\bar{\nu}_e$  survival probability we are interested in.

For  $\Delta m_{\odot}^2 \lesssim 1.0 \times 10^{-4} \text{ eV}^2$  and sufficiently small values of  $\sin^2 \theta$ ,  $\Delta m_{31}^2$  coincides effectively with  $\Delta m_{atm}^2$  of the two-neutrino  $\nu_\mu$  and  $\bar{\nu}_\mu$  oscillation analyses of the SK atmospheric neutrino data. If  $\sin^2 \theta > 0.01$ , a three-neutrino oscillation analysis of the atmospheric neutrino and CHOOZ data, performed under the assumption of  $\Delta m_{\odot}^2 \lesssim 1.0 \times 10^{-4} \text{ eV}^2$  [28], gives regions of allowed values of  $\Delta m_{atm}^2 = \Delta m_{31}^2$ , which are correlated with the value of  $\sin^2 \theta$ . The latter must satisfy the CHOOZ and Palo Verde constraints.

At present, as we have already indicated, a complete three-neutrino oscillation analysis of the atmospheric neutrino and CHOOZ data with  $\Delta m_{\odot}^2$  allowed to take values up to  $\sim (6.0 - 7.0) \times 10^{-4} \text{ eV}^2$ , i.e., in the region where deviations from the two-neutrino approximation could be non-negligible, is lacking in the literature. Therefore in what follows we will use representative values of  $\Delta m_{31}^2$  which lie in the region given by eq. (1).

### 3 The Difference between $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$

Let us discuss next in greater detail the difference between the  $\bar{\nu}_e$  surviving probabilities in the two cases of neutrino mass spectrum of interest,  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and  $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ . While the terms in the first two lines in eqs. (9) and (13) describe oscillations in  $L/E_\nu$  with frequencies  $\Delta m_{31}^2/4\pi$  and  $\Delta m_{\odot}^2/4\pi$ , respectively, the third term has the shape of beats, being produced by the interference of two waves, with the same amplitude but slightly different frequencies:

$$\begin{aligned} \cos\left(\frac{\Delta m_{31}^2 L}{2 E_\nu} - \frac{\Delta m_{\odot}^2 L}{2 E_\nu}\right) - \cos\frac{\Delta m_{31}^2 L}{2 E_\nu} &= 2 \sin\frac{\Delta m_{\odot}^2 L}{4 E_\nu} \sin\left(\frac{\Delta m_{31}^2 L}{2 E_\nu} - \frac{\Delta m_{\odot}^2 L}{4 E_\nu}\right) \\ &\simeq 2 \sin\frac{\Delta m_{\odot}^2 L}{4 E_\nu} \sin\left(\frac{\Delta m_{31}^2 L}{2 E_\nu}\right) \end{aligned} \quad (14)$$

This is a modulated oscillation with approximately the same frequency of the first term in eqs. (9) and (13) ( $\Delta m_{31}^2/4\pi$ ) and amplitude oscillating between 0 and  $2 \sin^2 \theta_{\odot}$  of the amplitude of the first term itself. The beat frequency is equal to the frequency of the dominant oscillation ( $\Delta m_{\odot}^2/4\pi$ ). The modulation is exactly in phase with the  $\Delta m_{\odot}^2$ -driven dominant oscillation of interest, so that the maximum of the oscillation amplitude of the interference term (third lines in the expressions for  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and  $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ ) is reached in coincidence with the points of maximal decreasing of the  $\bar{\nu}_e$  survival probability, where  $\Delta m_{\odot}^2 L/4 E = \pi/2$ , and vice versa - this amplitude vanishes at the local maxima of the survival probability. At the minima of the  $\bar{\nu}_e$  survival probability, for instance at  $\Delta m_{\odot}^2 L/4 E_\nu = \pi/2$ ,  $P^{NH(IH)}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  takes the value:

$$\begin{aligned} P^{NH(IH)}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \Big|_{\frac{\Delta m_{\odot}^2 L}{2\pi E_\nu} = 1} &= 1 - 2 \sin^2 \theta \cos^2 \theta - \cos^4 \theta \sin^2 2\theta_{\odot} \\ &\stackrel{(+)}{-} \cos 2\theta_{\odot} 2 \sin^2 \theta \cos^2 \theta \cos \pi \frac{\Delta m_{31}^2}{\Delta m_{\odot}^2}. \end{aligned} \quad (15)$$

From eqs. (9), (13) and (15) one deduces that:

- for maximal mixing,  $\cos 2\theta_{\odot} = 0$ , the last term cancels, and  $P^{NH} = P^{IH}$ ;
- for very small mixing angles,  $\cos 2\theta_{\odot} \simeq 1$ , the terms describing the oscillations driven by  $\Delta m_{31}^2$  in the NH and IH cases have opposite signs: the two waves are exactly out of phase.

- for intermediate values of  $\cos 2\theta_\odot$  from the LMA MSW solution region,  $\cos 2\theta_\odot \cong (0.3 - 0.6)$ , the  $\Delta m_{31}^2$ –driven contributions in the cases of normal and inverted hierarchy have still opposite signs and the magnitude of the effect is proportional to  $2 \cos 2\theta_\odot \sin^2 \theta$ .

The net result of these properties is that in the region of the minima of the  $\bar{\nu}_e$  survival probability due to  $\Delta m_\odot^2$ , where  $\Delta m_\odot^2 L / (2E) = \pi(2k+1)$ ,  $k = 0, 1, \dots$ , the difference between  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and  $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  is maximal. In contrast, at the maxima of  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  and  $P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  determined by  $\Delta m_\odot^2 L / (2E) = 2\pi k$ , we have, for any  $\sin^2 \theta_\odot$ ,  $P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ .

The two-neutrino oscillation approximation used in the analysis of the CHOOZ and Palo Verde data is rather accurate as long as  $\Delta m_\odot^2$  is sufficiently small [26]: for  $\Delta m_\odot^2 \lesssim 10^{-4} \text{ eV}^2$ , the  $L/E_\nu$  values characterizing these experiments, chosen to ensure maximal sensitivity to  $\Delta m_{31}^2 \gtrsim 10^{-3} \text{ eV}^2$ , are much smaller than the value at which the first minimum of  $P_{NH(IH)}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  due to the  $\Delta m_\odot^2$ -dependent oscillating term occurs. Correspondingly, the effect of the interference term is strongly suppressed by the beats. For  $\Delta m_\odot^2 \gtrsim 2 \times 10^{-4} \text{ eV}^2$  this is no longer valid and the interference term under discussion has to be taken into account in the analyses of the CHOOZ and Palo Verde data [26].

## 4 Measuring Large $\Delta m_\odot^2$ at Reactor Facilities

As is well-known, nuclear reactors are intense sources of low energy  $\bar{\nu}_e$  ( $E_\nu \lesssim 8 \text{ MeV}$ ), emitted isotropically in the  $\beta$ -decays of fission products with high neutron density [29]. Anti-neutrinos can then be detected through the positrons produced by inverse  $\beta$ -decay on nucleons. The reactor  $\bar{\nu}_e$  energy spectrum has been accurately measured and is theoretically well understood<sup>3</sup> [30]: it essentially consists of a bell-shaped distribution in energy centered around  $E_\nu \sim 4 \text{ MeV}$ , having a width of approximately 3 MeV. CHOOZ, Palo Verde and KamLAND are examples of experiments with reactor  $\bar{\nu}_e$ , the main difference being the distance between the source and the detector explored ( $L \sim 1 \text{ km}$  for CHOOZ and Palo Verde, and  $L \sim 180 \text{ km}$  for KamLAND).

The best sensitivity to a given value of  $\Delta m_\odot^2$  of the experiment of interest is at  $L$  at which the maximum reduction of the survival probability is realized. As can be seen from eqs. (9) - (13), this happens for  $L$  around  $L^* \equiv 2\pi E_\nu / \Delta m_\odot^2$ . This implies that for  $E_\nu = 4 \text{ MeV}$ , the optimal length to test neutrino oscillations with reactor experiments is:

$$L^* \cong \frac{5 \times 10^{-3}}{(\Delta m_\odot^2 / \text{eV}^2)} \text{ km} \quad (16)$$

The best sensitivity of KamLAND, for instance, is in the range of  $2 \div 3 \times 10^{-5} \text{ eV}^2$ . We will discuss next in greater detail the distances  $L$  which could be used to probe the LMA MSW solution region at  $\Delta m_\odot^2 > 10^{-4} \text{ eV}^2$ , in order to extract  $\Delta m_\odot^2$  from these oscillation experiments.

### 4.1 Total Event Rate Analysis

One of the signatures of the  $\bar{\nu}_e$ –oscillations would be a substantial reduction of the measured total event rate due to the reactor  $\bar{\nu}_e$  in comparison with the predicted one in the absence of oscillations. In order to compute the expected total event rate one has to integrate the  $\bar{\nu}_e$  survival probability multiplied by the  $\bar{\nu}_e$  energy spectrum over  $E_\nu$ . In Fig. 1 we show this averaged survival probability for different values of  $L$  as a function of  $\Delta m_\odot^2$ , using the “best fit” values [1, 5, 6, 7] for  $\Delta m_{31}^2$  and  $\sin^2 2\theta_\odot$ . When averaging over the  $\bar{\nu}_e$  energy spectrum, oscillatory effects with too short a period are washed out, and

<sup>3</sup>By reactor  $\bar{\nu}_e$  energy spectrum we mean here and in what follows the product of the  $\bar{\nu}_e$  production spectrum and the inverse  $\beta$ -decay cross-section, which gives the “detected” neutrino spectrum in the no oscillation case. The  $\bar{\nu}_e$  production spectrum is known with larger uncertainties at  $\bar{\nu}_e$  energies  $E_\nu \lesssim 2 \text{ MeV}$ , but this range is not of interest due to the threshold energy  $E_\nu^{th} \cong 1.8 \text{ MeV}$  of the inverse  $\beta$ -decay reaction [31]. Certain known time dependence at the level of a few percent is also present up to 3.5 MeV [32] and should possibly be taken into account in the analysis of the experimental data.

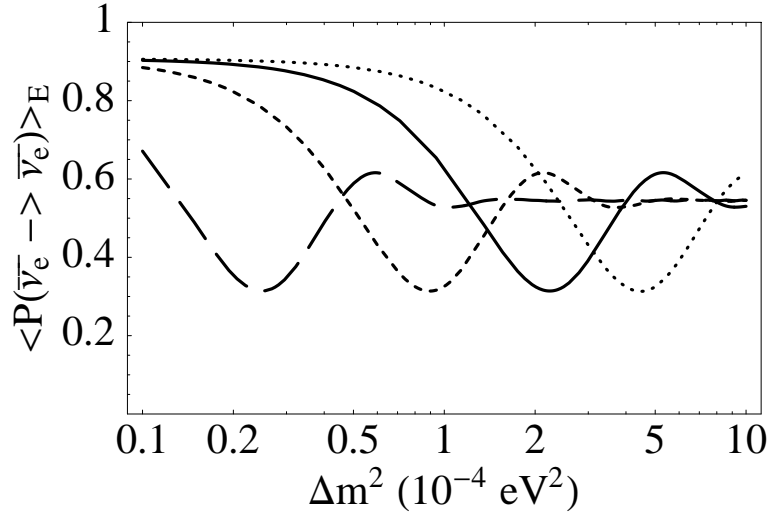


Figure 1: The reactor  $\bar{\nu}_e$  survival probability, averaged over the  $\bar{\nu}_e$  energy spectrum, for  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_\odot = 0.8$ ,  $\sin^2 \theta = 0.05$ , as a function of  $\Delta m_\odot^2$ . The curves correspond to  $L = 180 \text{ km}$  (long dashed),  $L = 50 \text{ km}$  (dashed),  $L = 20 \text{ km}$  (thick) and  $L = 10 \text{ km}$  (dotted), respectively.

the experiment is sensitive only to the average amplitude. This happens when the width  $\delta E_\nu$  of the energy spectrum is such that the integration runs over more than one period, i.e., approximately for:

$$\delta E_\nu \gtrsim \frac{4\pi E_\nu^2}{\Delta m^2 L} \simeq \frac{4 \times 10^4 \text{ eV}^3}{\Delta m^2 (L/Km)}. \quad (17)$$

Since  $\delta E_\nu \sim 3 \text{ MeV}$ , at KamLAND this happens approximately for  $\Delta m_\odot^2 \gtrsim 7 \times 10^{-5} \text{ eV}^2$ . The corresponding curve in Fig. 1 indicates that the actual sensitivity extends to somewhat larger values of  $\Delta m_\odot^2$  than what is expected on the basis on the above estimate, but the total event rate becomes flat for  $\Delta m_\odot^2 \gtrsim 10^{-4} \text{ eV}^2$ . This means that KamLAND will be able, through the measurement of the total even rate, to test all the region of the LMA MSW solution and determine whether the latter is the correct solution of the solar neutrino problem, but will provide a precise measurement of  $\Delta m_\odot^2$  only if  $\Delta m_\odot^2 \lesssim 10^{-4} \text{ eV}^2$ . If  $\Delta m_\odot^2 \gtrsim 2 \times 10^{-4} \text{ eV}^2$ , it would be possible to obtain only a lower bound on  $\Delta m_\odot^2$  and a new experiment might be required to determine  $\Delta m_\odot^2$ . Fig. 1 shows that as  $L$  decreases, the sensitivity region moves to larger  $\Delta m_\odot^2$ . These results imply that a reactor  $\bar{\nu}_e$  experiment with  $L \cong 20 \text{ km}$  can probe the range

$$0.8 \times 10^{-4} \text{ eV}^2 < \Delta m_\odot^2 \lesssim 6 \times 10^{-4} \text{ eV}^2. \quad (18)$$

For  $L = 20 \text{ km}$  one finds that the best sensitivity is for  $\Delta m_\odot^2 \cong 2 \times 10^{-4} \text{ eV}^2$ , but the predicted event rate deviates from being flat actually for  $\Delta m_\odot^2$  as large as <sup>4</sup>  $\sim 6 \times 10^{-4} \text{ eV}^2$ .

In order to have a precise measure of  $\Delta m_\odot^2$  in the case of the largest values given in eq. (2),  $\Delta m_\odot^2 \cong (7 \div 8) \times 10^{-4} \text{ eV}^2$  one should use  $L \cong 10 \text{ km}$ . However, with such a short baseline the experiment would hardly be sensitive to  $\Delta m_\odot^2 \cong (1 \div 3) \times 10^{-4} \text{ eV}^2$ , which might actually be the most interesting region to be explored after KamLAND.

Applying eq. (17) with  $\Delta m^2 = \Delta m_{31}^2$ , one sees that for the ranges of  $L$  which allow to probe  $\Delta m_\odot^2$  from the LMA MSW solution region, the total event rate is not sensitive to the oscillations driven

<sup>4</sup>The fact that if  $\Delta m_\odot^2 \cong 2 \times 10^{-4} \text{ eV}^2$  a reactor  $\bar{\nu}_e$  experiment with  $L \cong 20 \text{ km}$  would allow to measure  $\Delta m_\odot^2$  with a rather good precision was also noticed recently in [33].



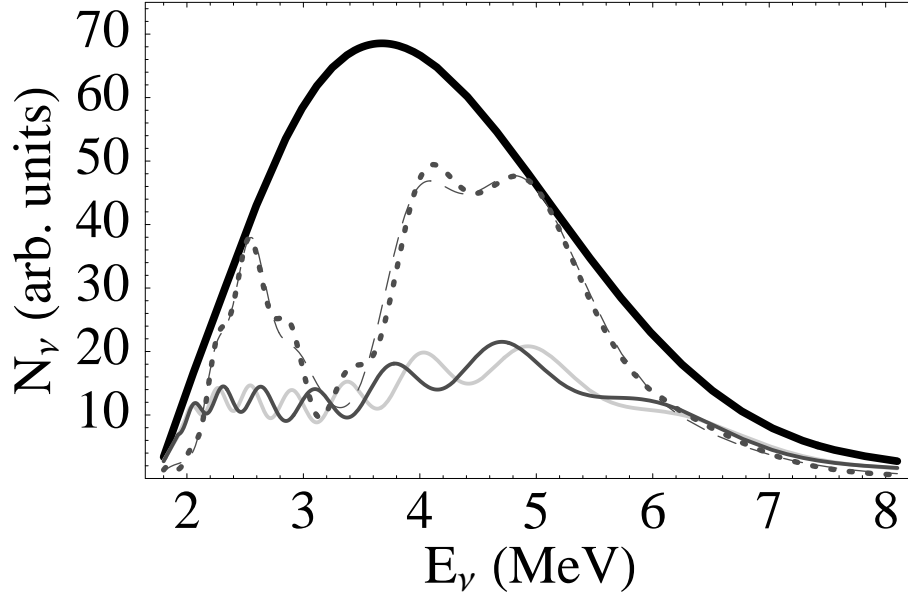


Figure 2: The reactor  $\bar{\nu}_e$  energy spectrum at distance  $L = 20$  km from the source, in the absence of  $\bar{\nu}_e$  oscillations (double-thick solid line) and in the case of  $\bar{\nu}_e$  oscillations characterized by  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_\odot = 0.8$  and  $\sin^2 \theta = 0.05$ . The thick lines are obtained for  $\Delta m_\odot^2 = 2 \times 10^{-4} \text{ eV}^2$  and correspond to NH (light grey) and IH (dark grey) neutrino mass spectrum. Shown is also the spectrum for  $\Delta m_\odot^2 = 6 \times 10^{-4} \text{ eV}^2$  in the NH (dotted) and IH (dashed) cases.

by  $\Delta m_{31}^2 \gtrsim 1.5 \times 10^{-3} \text{ eV}^2$ . Thus, the total event rate analysis would determine  $\Delta m_\odot^2$  which would be the same for both the normal and inverted hierarchy neutrino mass spectrum.

#### 4.2 Energy Spectrum Distortions

An unambiguous evidence of neutrino oscillations would be the characteristic distortion of the  $\bar{\nu}_e$  energy spectrum. This is caused by the fact that, at fixed  $L$ , neutrinos with different energies reach the detector in a different oscillation phase, so that some parts of the spectrum would be suppressed more strongly by the oscillations than other parts. The search for distortions of the  $\bar{\nu}_e$  energy spectrum is essentially a direct test of the  $\bar{\nu}_e$  oscillations. It is more effective than the total rate analysis since it is not affected, e.g., by the overall normalization of the reactor  $\bar{\nu}_e$  flux. However, such a test requires a sufficiently high statistics and sufficiently good energy resolution of the detector used.

Energy spectrum distortions can be studied, in principle, in an experiment with  $L = 20$  km. In Fig. 2 we show the comparison between the  $\bar{\nu}_e$  spectrum expected for  $\Delta m_\odot^2 = 2 \times 10^{-4} \text{ eV}^2$  and  $\Delta m_\odot^2 = 6 \times 10^{-4} \text{ eV}^2$  and the spectrum in the absence of  $\bar{\nu}_e$  oscillations. No averaging has been performed and the possible detector resolution is not taken into account. The curves show the product of the probabilities given by eqs. (9) and (13) and the predicted reactor  $\bar{\nu}_e$  spectrum [34].

As Fig. 2 illustrates, the  $\bar{\nu}_e$  spectrum in the case of oscillation is well distinguishable from that in the absence of oscillations. Moreover, the shape of the spectrum exhibits a very strong dependence on the value of  $\Delta m_\odot^2$ . A likelihood analysis of the data could be able to determine the value of  $\Delta m_\odot^2$  with a rather good precision. If the energy bins used in the measurement of the spectrum are sufficiently large, the value of  $\Delta m_\odot^2$  thus determined should coincide with value obtained from the analysis of the total event rate and should be independent of  $\Delta m_{31}^2$ .

## 5 Normal vs. Inverted Hierarchy

In Fig. 2 we show the deformation of the reactor  $\bar{\nu}_e$  spectrum both for the normal and inverted hierarchy neutrino mass spectrum: as long as no integration over the energy is performed, the deformations in the two cases of neutrino mass spectrum can be considerable, and the sub-leading oscillatory effects driven by the atmospheric mass squared difference (see the first and the third line of eqs. (9) - (13)) can, in principle, be observed. They could be used to distinguish between the two hierarchical patterns, provided the solar mixing is not maximal,  $\sin^2 \theta$  is not too small and  $\Delta m_{31}^2$  is known with high precision. It should be clear that the possibility we will be discussing next poses remarkable challenges.

The experiment under discussion could be in principle an alternative to the measurement of the sign of  $\Delta m_{31}^2$  in long (very long) baseline neutrino oscillation experiments [21, 22, 23] or in the experiments with atmospheric neutrinos (see, e.g., [24]).

The magnitude of the effect of interest depends, in particular, on three factors, as we have already pointed out:

- the value of the solar mixing angle  $\theta_\odot$ : the different behavior of the two survival probabilities is due to the difference between  $\sin^2 \theta_\odot$  and  $\cos^2 \theta_\odot$ ; correspondingly, the effect vanishes for maximal mixing; thus, the more the mixing deviates from the maximal the larger the effect;
- the value of  $\sin^2 \theta$ , which controls the magnitude of the sub-leading effects due to  $\Delta m_{31}^2$  on the  $\Delta m_\odot^2$ -driven oscillations: the effect of interest vanishes in the decoupling limit of  $\sin^2 \theta \rightarrow 0$ ;
- the value of  $\Delta m_\odot^2$  (see Fig. 1): for given  $L$  and  $\Delta m_\odot^2$  the difference between the spectrum in the cases of normal and inverted hierarchy is maximal at the minima of the survival probability, and vanishes at the maxima.

A rough estimate of the possible difference between the predictions of the event rate spectrum for the two hierarchical patterns, is provided by the ratio between the difference and the sum of the two corresponding probabilities at  $\Delta m_\odot^2 L = 2\pi E_\nu$ :

$$\frac{P_{NH} - P_{IH}}{P_{NH} + P_{IH}} = \frac{2 \cos 2\theta_\odot \sin^2 \theta \cos^2 \theta}{1 - 2 \sin^2 \theta \cos^2 \theta - \cos^4 \theta \sin^2 2\theta_\odot} \cos \pi \frac{\Delta m_{31}^2}{\Delta m_\odot^2}. \quad (19)$$

The ratio could be rather large: the factor in front of the  $\cos \pi \Delta m_{31}^2 / \Delta m_\odot^2$  is about 25% for  $\sin^2 2\theta_\odot = 0.8$  and  $\sin^2 \theta = 0.05$ .

The actual feasibility of the study under discussion depends crucially on the integration over (i.e., the binning in) the energy: for the effect not to be strongly suppressed, the energy resolution of the detector  $\Delta E_\nu$  must satisfy:

$$\Delta E_\nu \lesssim \frac{4\pi E_\nu^2}{\Delta m_{31}^2 L} \simeq \frac{2 \div 6 \times 10^4 \text{ eV}^3}{\Delta m_{31}^2 (L/\text{km})}. \quad (20)$$

For small distances ( $\sim 1$  km) this condition could be satisfied for  $\delta E_\nu \simeq \Delta E_\nu$ , but at  $L = 20$  Km, for  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$  and  $E_\nu$  around 2.5 MeV, one should have  $\Delta E_\nu \lesssim 0.5$  MeV.

Our discussion so far was performed for simplicity in terms of the reactor  $\bar{\nu}_e$  energy spectrum, while in the experiments of interest one measures the energy of the positron emitted in the inverse  $\beta$ -decay,  $E_e$ . The relation between  $E_e$  and  $E_\nu$  is well known (see for instance [34]), and, up to corrections of at most few per cent, consists just in a shift due to the threshold energy of the process:  $E_\nu \cong E_e + E_\nu^{th}$ . The maximal  $\Delta E_\nu$  allowed in order to make the effect observable can be then directly compared to the experimental positron energy resolution  $\Delta E_e$  <sup>5</sup>.

<sup>5</sup>In the CHOOZ experiment, for instance, the energy resolution was  $\Delta E_e \simeq 0.40$  MeV [11]. KamLAND is expected to have a resolution  $\Delta E_e/E_e = 10\%/\sqrt{E_e}$ , where  $E_e$  is in MeV [35].

For  $\Delta m_{\odot}^2 \lesssim 10^{-4} \text{ eV}^2$ , the first (most significant) minimum of the survival probability can be explored if  $L \sim 180 \text{ km}$ . In this case, due to the bigger distance  $L$ , the energy resolution required would be by a factor of ten smaller. This means that for  $\Delta m_{\odot}^2 \ll \Delta m_{31}^2$ , it is practically impossible to realize the condition of maximization of the difference between the survival probabilities in the two cases of neutrino mass spectrum without strongly suppressing the magnitude of the difference by the binning of the energy spectrum.

In order to illustrate what are the concrete possibilities in the case of the experiment under discussion, we have divided the energy interval  $2.7 \text{ MeV} < E_\nu < 7.2 \text{ MeV}$  into 15 bins, with  $\Delta E_\nu = 0.3 \text{ MeV}$ , and calculated the value of the product of the survival probability and the energy spectrum in each of the bins. The results are shown in Fig. 3.

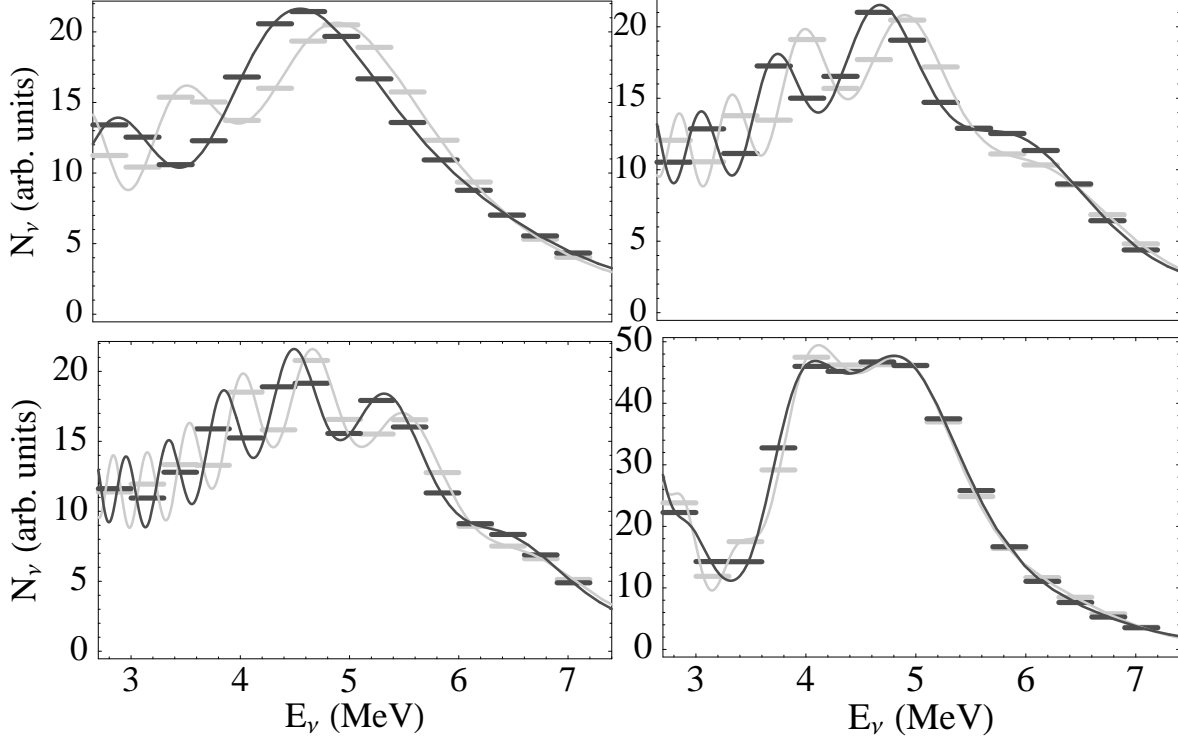


Figure 3: Comparison between the predicted event rate spectrum at  $L = 20 \text{ km}$ , measured in energy bins having a width of  $\Delta E_\nu = 0.3 \text{ MeV}$  in the cases of normal (light grey) and inverted (dark grey) neutrino mass hierarchy. The two upper and the lower left figures are for  $\Delta m_{\odot}^2 = 2 \times 10^{-4} \text{ eV}^2$ ,  $\sin^2 2\theta_{\odot} = 0.8$ ,  $\sin^2 \theta = 0.05$ , and  $\Delta m_{31}^2 = 1.3; 2.5; 3.5 \times 10^{-3} \text{ eV}^2$ , respectively. The lower right figure was obtained for  $\Delta m_{\odot}^2 = 6 \times 10^{-4} \text{ eV}^2$  and  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ .

It is clear from Fig. 3 that for  $\Delta m_{31}^2 \cong (1.5 - 3.0) \times 10^{-3} \text{ eV}^2$ ,  $\Delta m_{\odot}^2 \cong 2.0 \times 10^{-4} \text{ eV}^2$ ,  $\sin^2 2\theta_{\odot} = 0.8$  and  $\sin^2 \theta = 0.05$  it might be possible to distinguish the two cases of neutrino mass spectrum by measurements of the energy spectrum in the experiment with reactor  $\bar{\nu}_e$  under discussion. If, however,  $\sin^2 \theta \lesssim 0.01$ , and/or  $\sin^2 2\theta_{\odot} \gtrsim 0.9$ , and/or  $\Delta m_{\odot}^2 \cong 6.0 \times 10^{-4} \text{ eV}^2$  while  $\Delta m_{31}^2 \cong (1.5 - 3.0) \times 10^{-3} \text{ eV}^2$ , the difference between the spectra in the two cases becomes hardly observable. Further, in obtaining Fig. 3 we have implicitly assumed that  $\Delta m_{31}^2$  is known with negligible uncertainty. Actually, for the difference between the spectra under discussion to be observable,  $\Delta m_{31}^2$  has to be determined, according to our estimates, with a precision of  $\sim 10\%$  or better: given the values of  $\Delta m_{\odot}^2$ ,  $\sin^2 2\theta_{\odot}$  and  $\sin^2 \theta$ , a spectrum in the NH case corresponding to a given  $\Delta m_{31}^2$  can be rather close in shape to the spectrum in the IH case for a different value of  $\Delta m_{31}^2$ . There is no similar effect when

varying  $\Delta m_{\odot}^2$ .

## 6 Conclusions

Reactor experiments have the possibility to test the LMA MSW solution of the solar neutrino problem. While the KamLAND experiment should be able to test this solution, a new experiment with a shorter baseline might be required to determine  $\Delta m_{\odot}^2$  with high precision if the results of the KamLAND experiment show that  $\Delta m_{\odot}^2 > 10^{-4} \text{ eV}^2$ . Performing a three-neutrino oscillation analysis we find that a value of  $10^{-4} \text{ eV}^2 < \Delta m_{\odot}^2 \lesssim 8.0 \times 10^{-4} \text{ eV}^2$  could be measured with a high precision in experiments with  $L \cong (20 - 10) \text{ km}$ ; a value of  $\Delta m_{\odot}^2$  in the interval  $\Delta m_{\odot}^2 \cong (1.0 - 6.0) \times 10^{-4} \text{ eV}^2$  can be determined with  $L \cong 20 \text{ km}$ . Furthermore, such an experiment might also be able to distinguish between the cases of neutrino mass spectrum with normal and inverted hierarchy. However, this possibility poses remarkable challenges and could be realized for a limited range of values of the relevant parameters. The corresponding detector must have a good energy resolution ( $\Delta E_e \lesssim 0.50 \text{ MeV}$ ), the mixing angle constrained by the CHOOZ and Palo Verde data  $\theta$  must be sufficiently large ( $\sin^2 \theta \sim 0.03 - 0.05$ ), and the “solar” mixing angle  $\theta_{\odot}$  should not be maximal ( $\sin^2 2\theta_{\odot} \lesssim 0.9$ ). In addition, the value of  $\Delta m_{31}^2$ , which is responsible for the dominant  $\nu_{\mu} \rightarrow \nu_{\tau}$  and  $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\tau}$  oscillations of the atmospheric neutrinos, should be known with a high precision. However, as it is well known, “only those who wager can win” [36].

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## References

- [1] Super-Kamiokande Collaboration, Y. Fukuda et al., *Phys. Rev. Lett.* **81** (1998) 1562.
- [2] Q. R. Ahmad et al. [SNO Collaboration], *Phys. Rev. Lett.* **87** (2001) 071301 (*nucl-ex/0106015*).
- [3] S. Fukuda et al. (Super-Kamiokande Collaboration), *Phys. Rev. Lett.* **86** (2001) 5651 (*hep-ex/0103032*).
- [4] V. Barger, D. Marfatia and K. Whisnant, *hep-ph/0106207*.
- [5] G. L. Fogli et al., *hep-ph/0106247*.
- [6] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, *JHEP* **0108** (2001) 014 (*hep-ph/0106258*).
- [7] A. Bandyopadhyay et al., *Phys. Lett.* **B519** (2001) 83. (*hep-ph/0106264*).
- [8] P. Creminelli, G. Signorelli and A. Strumia, *hep-ph/0102234*, version 2 from 10 July, 2001.
- [9] M.V. Garzelli and C. Giunti, *hep-ph/0108191*.
- [10] P. I. Krastev and A. Y. Smirnov, *hep-ph/0108177*.
- [11] M. Apollonio et al. [CHOOZ Collaboration], *Phys. Lett.* **B466** (1999) 415 (*hep-ex/9907037*).
- [12] F. Boehm, J. Busenitz et al., *Phys. Rev. Lett.* **84** (2000) 3764 and *Phys. Rev. D* **62** (2000) 072002.
- [13] MINOS Collaboration, P. Adamson et al., NuMI-L-476 (March 1999); Web page: <http://www.hep.anl.gov/ndk/hypertext/numi.html>.
- [14] A. Piepke [KamLAND Collaboration], *Nucl. Phys. Proc. Suppl.* **91** (2001) 99.

- [15] S.M. Bilenky et al., *Phys. Lett. B* **356** (1995) 273 and *Phys. Rev. D* **54** (1996) 1881.
- [16] S. M. Bilenky, C. Giunti and W. Grimus, *Prog. Part. Nucl. Phys.* **43** (1999) 1 (hep-ph/9812360).
- [17] B. Pontecorvo, *Zh. Eksp. Teor. Fiz.* **33** (1957) 549, and **34** (1958) 247.
- [18] Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theor. Phys.* **28** (1962) 870.
- [19] S.M. Bilenky and S.T. Petcov, *Rev. Mod. Phys.* **59** (1987) 671.
- [20] S. M. Bilenky, S. Pascoli and S. T. Petcov, *Phys. Rev. D* **64** (2001) 053010 (hep-ph/0102265).
- [21] P. Lipari, *Phys. Rev. D* **61**, 113004 (2000).
- [22] V. Barger et al., *Phys. Rev. D* **62** (2000) 013004.
- [23] M. Freund et al., *Nucl. Phys.* **B578** (2000) 27.
- [24] M.C. Banuls, G. Barenboim and J. Bernabeu, *Phys. Lett. B* **531** (2001) 391; J. Bernabeu et al., hep-ph/0110071
- [25] S.M. Bilenky et al., *Phys. Rev. D* **54** (1996) 4432 (hep-ph/9604364).
- [26] S. M. Bilenky, D. Nicolo & S. T. Petcov, Ref. SISSA 63/2001/EP, TUM-HEP-427/01, to appear.
- [27] G. L. Fogli, E. Lisi and A. Palazzo, hep-ph/0105080.
- [28] M. C. Gonzalez-Garcia et al., *Phys. Rev. D* **63** (2001) 033005 (hep-ph/0009350).
- [29] B. Pontecorvo, Chalk River Lab. Report PD-205 (1946).
- [30] see for reviews: G. Gratta, *Nucl. Phys. Proc. Suppl.* **85**, 72 (2000) (hep-ex/9905011); and C. Bemporad, G. Gratta and P. Vogel, hep-ph/0107277.
- [31] V. I. Kopeikin, hep-ph/0110030.
- [32] V. I. Kopeikin, L. A. Mikaelyan and V. V. Sinev, *Phys. Atom. Nucl.* **64**, 849 (2001) [*Yad. Fiz.* **64**, 914 (2001)] (hep-ph/0110290).
- [33] A. Strumia and F. Vissani, hep-ph/0109172.
- [34] See, e.g., C. Bemporad, G. Gratta and P. Vogel in [30]
- [35] M. C. Gonzalez-Garcia and C. Pena-Garay, hep-ph/0111432, and references therein.
- [36] W. Pauli, Letter to Participants of a Physics Meeting in Tübingen (H. Geiger, L. Meitner, etc.), Germany, December 4, 1930.